

ENERGY LOSS FROM CLOSED COMBUSTION SYSTEMS

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The process of combustion in a closed system, such as a cylinder in a piston engine, is executed in three steps: (1) ignition, devoid of any pressure change; (2) the dynamic stage, manifested by distinct pressure rise; and (3) decay, associated with pressure drop. This paper is concerned solely with the second step, exhibiting the dynamic behavior of the system. The effectiveness of the exothermic energy generated in a closed system is affected only by the loss incurred as a consequence of heat transfer to the walls. Its evaluation is, then, the principal subject of the reported study. Measurements of heat transfer to the walls were made for this purpose in a constant volume vessel by the use of thin film thermometers, simultaneously with pressure and schlieren records. Profiles of energy loss were determined on the basis of a self-similarity theory developed on the basis of the self-similitude exhibited by heat flux profiles. At the same time, the mass fraction of fuel consumed in the course of the dynamic stage of combustion was deduced from the measured pressure trace by thermodynamic analysis based on the balances of mass, volume, and energy. A power-law correlation was thereby derived between the fuel consumed solely to elevate pressure, referred to as effective, and its total amount. As a phase relationship, this correlation is valid irrespectively of the geometry of the enclosure, as well as of its variation in a piston engine, especially in the vicinity of the top dead center where the dynamic stage of combustion takes place. Its utility is then quite general in permitting the ineffective part of consumed fuel, associated with energy lost by heat transfer to the walls, to be evaluated from the pressure record, without taking into account the geometry of the enclosure or its deformation.

Background

The process of combustion in a closed system, such as a cylinder of a piston engine, is executed in three steps:

1. Ignition, undetectable by a pressure sensor at the wall of the enclosure
2. The dynamic stage, manifested by distinct pressure rise
3. Decay dominated by heat transfer to the walls, associated with pressure drop

The second step, with which this paper is primarily concerned, is, in turn, manifested in two ways: (1) dynamic, exhibited by pressure rise—the mechanical potential for useful work, and (2) thermal, associated with the energy loss incurred by heat transfer to the walls.

Heat transfer in piston engines has been, of course, a subject of extensive research (see, e.g., Refs. [1–5]). Especially prominent in this field are the multifarious publications of Woschni [6–19]. Of particular significance to our studies were the contributions of Greif and his associates [20–26]. The self-similarity theory reported here was developed

on their basis, allowing the energy lost by heat transfer to the walls in the course of combustion to be evaluated from a measured pressure record.

Most of the research in this field has been addressed to the determination of the heat flux, as eminently appropriate for the subject of heat transfer. For thermodynamic analysis, however, it is not the rate that matters, but the amount. The major advantage of the solution obtained here is its ability to deduce the evolution of the energy lost by heat transfer to the walls, κ , from the pressure transducer record that, for an engine, is expressed in terms of an indicator diagram. On this basis one can determine the evolution of mass (or volume) fraction burned (equivalent to the fraction of products at equilibrium), as well as that of the mass averaged specific volumes and, hence, the temperatures.

Experiments

To accomplish this task, an experimental study was carried out. The experimental vessel used was a closed cylinder 3.5 in. in diameter and 2 in. deep, amounting to 283 cm³ in volume. Its size corresponds to that of a Committee on Fuel Research (CFR) engine cylinder at a compression ratio of 8:1,

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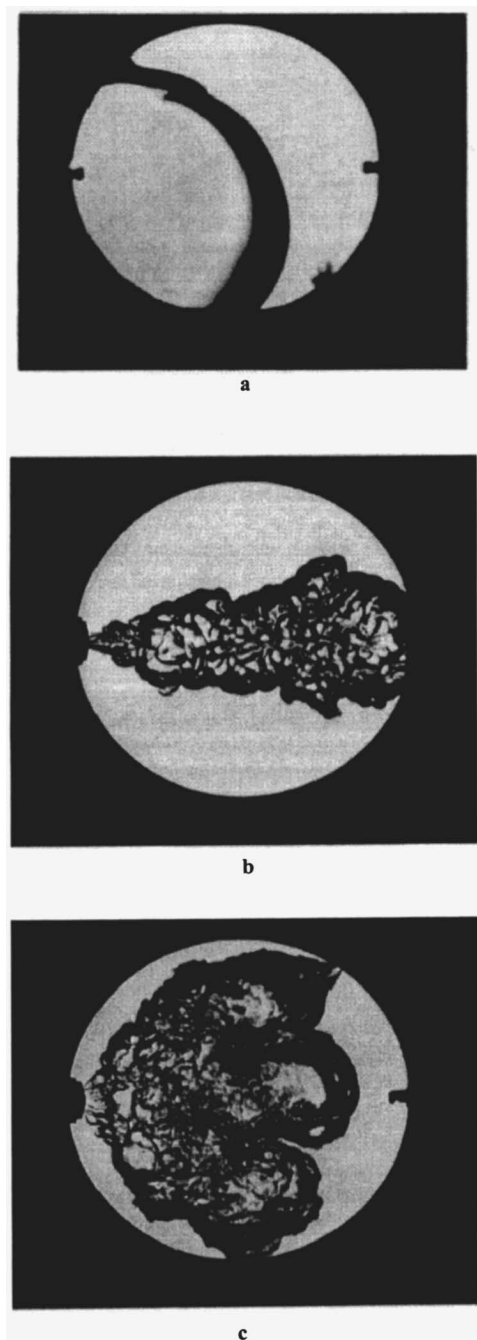


FIG. 1. Schlieren records of combustion fields. (a) Case F: Flame traversing the charge; (b) Case S: Single stream flame jet; (c) Case T: Triple stream flame jet.

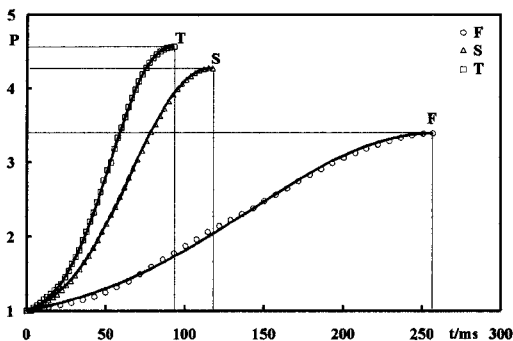


FIG. 2. Normalized pressure data and their life functions.

when the piston is at a 60-degree crank angle from the top dead center. For the tests the cylinder was filled with a carefully premixed propane/air mixture at an equivalence ratio of 0.6, initially at a pressure of 5 bar and a temperature of 65 °C.

Heat flux measurements were made by the use of thin film heat transfer gauges, simultaneously with pressure transducer recording associated with high-speed schlieren cinematography taken through optical plates closing the cylinder at both ends. By the use of a variety of ignition systems, the experiments covered a wide range of operating conditions, from an apparently laminar flame to a fully developed turbulent combustion. This was achieved by the use of three distinct modes of executing the exothermic process: (1) spark-ignited flame traversing the charge, F, (2) single stream flame jet (referred to previously as PJC (pulsed jet combustion) or PCJ (pulsed combustion jet) [27,28,29]) initiated combustion, S, and (3) triple-stream flame jet combustion, T. A detailed report on this study is outside the scope of this paper.

The three modes of combustion are depicted by schlieren records of Figs. 1a, b, and c. In mode F, the flame front was clearly delineated, acting as the boundary between the reactants and products. Mode S appeared as a round turbulent jet plume, while mode T was disk-shaped, in compliance with the geometry of the enclosure. As evident on the photographs, in the first case the products were in contact with the walls right from the outset. In the second, the combustion zone reached the walls upon a distinct time delay, while in the third the delay was longer.

Concomitantly, the measured pressure traces are presented by data points in Fig. 2 in terms of their normalized values, $P(t) \equiv p(t)/p_i$, together with their life functions [30]. At the same time, the temperature profiles, $T(t)$, were sensed by thin film resistance thermometers placed in two strategic locations at the walls of the cylindrical enclosure, one at the side and the other at the back, across from the point of ignition or the jet influx.

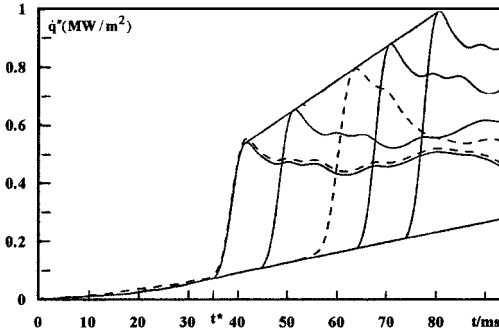


FIG. 3. Self-similar features of heat flux profiles.

Interpretation

Heat Transfer

The heat flux was evaluated from the temperature profile, $T(t)$, by the Duhamel superposition integral [31]

$$\dot{q}''(t) = \beta \int_0^t \frac{dT(t)}{dt_i} \frac{dt_i}{(t - t_i)^{1/2}} \quad (1)$$

where $\beta = (k\rho c/\pi)^{1/2}$, while k is the thermal conductivity of the substrate, ρ its density, and c the specific heat. For Macor, we employed, at the temperature of 300 K, $k = 12.87$ MJ/(s cm K), $\rho = 2.52$ g/cm³, and $c = 0.795$ J/(g K), so that $\beta = 2.866$ MJ/(s^{1/2} cm² K).

The integration was performed using the algorithm of Arpaci [32]. The heat flux profiles, thus determined, displayed a remarkably similar pattern: a ramp followed by a sharp pulse with a wavy decay, as depicted in Fig. 3. The initial rise is, according to our estimates, caused predominantly by radiation from the high-temperature products, replete with such strong radiators as H₂O and CO₂, while the walls are in contact with the reactants which are, nonetheless, heated by compression due to expansion of the products. The high amplitude pulse is attributed, of course, to convection and conduction from the exothermic zone as it “touches” (operationally rather than factually) the wall—the event starting at a critical time instant, t^* . This furnished a fundamental reason for the introduction of a self-similarity theory, according to which the heat flux profiles could be subsequently integrated to evaluate the total amount of energy lost by heat transferred to the walls. This was accomplished as follows.

The evaluation of heat transferred to the walls involves double integration

$$q_{wf}(t) = \int_0^t \int_0^{A_f} \dot{q}''(A, t_i) dA dt_i \quad (2)$$

where subscript f denotes the final state of the combustion event when the exothermic zone extends over the total wall area of the enclosure.

The evolution of heat transfer from a combustion event to the walls of the enclosure consists of two stages.

Up to $t = t^*$, when the exothermic zone first comes into contact with the wall at $A = A^*$, as the non-uniform distribution of radiative flux can be disregarded, while only the reactants are in contacts with the wall

$$q_{w1}(t) = A_f \int_0^{t^*} \dot{q}''_{w1}(t_i) dt_i \quad (3)$$

After that, both the reactants and the products touch the wall. For the part of the wall remaining in touch with the reactants

$$q_{w2}(t) = \int_{t^*}^t [A_f - A(\tau)] \dot{q}''_2(\tau) d\tau \quad (4)$$

while heat transfer to the wall in contact with the products is expressed in terms of a full Duhamel integral

$$q_{w3}(t) = A_f \int_{t^*}^t \left[\int_0^\tau \dot{q}''_3(t_j; t_i - t_j) \frac{dA(t_i)}{dt_j} dt_j \right] d\tau \quad (5)$$

where $A(t_j) \equiv A(t_j)/A_f$, while, with reference to the geometrical features of the self-similar profiles displayed in Fig. 3,

$$\begin{aligned} \dot{q}''_3(t_j; t_i - t_j) &= (t_j - t^*) A(t_j) \dot{q}''_3(t^*; t_i - t^*) \\ &\quad + [\dot{q}''_2(t_j) - \dot{q}''_1(t^*)] \end{aligned} \quad (6)$$

To evaluate \dot{q}''_3 from the above, knowledge of the functional relationship $A(t_j)$ is required. This can be inferred only from experimental insight, which is here provided by the schlieren records. Obtained thereby are, *per force*, just fragmentary estimates, since the wall area in contact with the products can be discerned only when the geometry of the exothermic front is relatively simple. In the literature this area has been referred to as that of a “burned-gas wetted wall,” and its determination, even in the simplest case of a hemispherical flame front, demanded a considerable amount of geometrical detail. We were able to reduce this effort significantly by observing that the fragmented data on $A(t_j)$ we obtained from the schlieren records are in quite a good agreement with the pressure profile, $P(t_j)$. We used, therefore, a fairly plausible approximation, according to which

$$A(t_j) = [P(t_j) - P^*]/[P_f - P^*] \quad (7)$$

where $P^* = P(t^*)$.

The total amount of heat transferred to the walls of the enclosure, q_w , was evaluated as a sum of integrals expressed by equations 3, 4, and 5, with equations 6 and 7 taken into account. The results, expressed in terms of the time profiles of energy loss, q/Wh , are presented in Fig. 4 by data points, together with curves derived from the thermodynamic

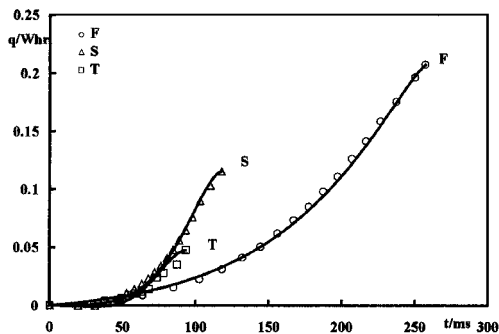


FIG. 4. Profiles of energy lost by heat transfer to the walls, q/Whr , deduced from heat transfer measurements, expressed by data points, in comparison to analytical results evaluated from pressure records, presented by curves.

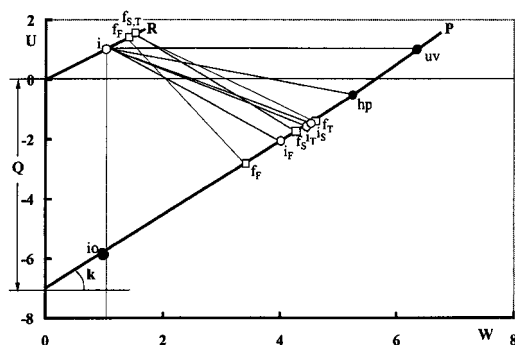


FIG. 5. Le Chatelier diagram.

analysis of the concomitantly recorded pressure traces, displayed in Fig. 2. The method used to produce them is described in the next section.

Thermodynamics

As pointed out at the outset, a closed combustion system is prominently dynamic in nature. This is reflected by its *thermokinetic* behavior, according to which the time profiles of its intrinsic parameters can be expressed in terms of the *life function* (32)

$$x_k = \frac{e^{\zeta} - 1}{e^{\zeta_f} - 1} \quad (8)$$

where, with respect to $\tau \equiv (t - t_i)/(t_f - t_i)$, $\zeta \equiv \alpha/\chi + 1 [1 - (1 - \tau)^{\chi+1}]$, while $k = P, F$. This expression is, in effect, a reverse of the so-called Wiebe [33–35]—alias of Vibe [36,37]—function.

For the pressure trace,

$$x_p = \frac{P - 1}{P_f - 1} \quad (9)$$

while, for the mass fraction of fuel consumed in the

course of the dynamic stage of combustion in a pre-mixed system, confined within an enclosure of constant volume,

$$x_F = \frac{1}{\eta_F} \frac{kP - (k - 1)P^{1-n^{-1}} - 1 + \kappa}{Q - (k - 1)P^{1-n^{-1}}} \quad (10)$$

where $\kappa \equiv q_w/c_v T_i$.

The above expression was derived from the *thermostatic* balances of mass, volume, and energy [38,39], while the *thermodynamic properties* of the reactants, R, and products, P, taken appropriately into account, are displayed on the Le Chatelier diagram in Fig. 5. The reactants are identified by a given composition of the charge, while the products are at its thermodynamic equilibrium. The coordinates of Fig. 5 are \bar{U} , the normalized internal energy, and \bar{W} , the normalized product of pressure and specific volume, both taken with respect to their initial conditions. Provided thus are the values of Q and k . The coefficient, η_F , expresses the efficiency of the use of fuel for the dynamic stage of combustion, that is, the ratio of fuel consumed in its course to that supplied to the system.

Correlation

The expression for x_F provided by equation 10 can be interpreted as a sum

$$x_F = x_p + x_q \quad (11)$$

subscript p denoting its effective part, specified by equation 10 for $\kappa = 0$, while q refers to the ineffective rest.

From the results presented in the next section a correlation was deduced, according to which the effective part of consumed fuel is related to its total amount by a power law:

$$x_{pn} \equiv \frac{x_p}{x_{pf}} = 1 - (1 - x_F)^\sigma \quad (12)$$

where $\sigma = 2 - x_{pf}^{3/2}$.

The numerator in the above expression is evaluated from equation 10, for $\kappa = 0$, while the denominator is given by $x_{pf} = x_{pmax}$. Thus, x_{pn} is expressed as an explicit function of P . The total mass fraction of fuel consumed by the exothermic process of combustion is then

$$x_F = 1 - (1 - x_{pn})^{1/\sigma} \quad (13)$$

Results

The outcome is presented in Fig. 6 in terms of the progress parameters for mass fractions of fuel consumed in the course of the dynamic stage of combustion, x_F , together with its effective and ineffective fractions, expressed by their life functions. Their parameters are displayed in Table 1. Profiles of the

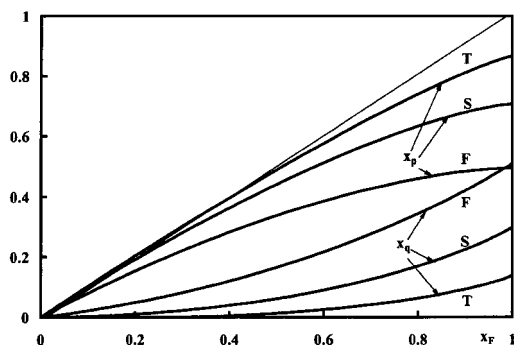


FIG. 6. Mass fractions of effective and ineffective parts of consumed fuel and its total amount.

TABLE 1

Parameters of dynamic stage in constant volume vessel

Case	F	S	T
T/ms	257	118	93
p _f /bar	16.95	21.35	22.80
P _f	3.39	4.27	4.56
Q		6.98	
k		1.23	
α_{II}	3.76	7.46	9.00
χ_{II}	0.85	1.22	1.32
α_F	2.57	5.73	7.90
χ_F	0.26	0.75	1.06
x_{pf}	0.49	0.71	0.86
η_F	0.87	0.83	0.74
α_v	3.27	3.73	3.64
η_v	0.96	0.95	0.91

normalized energy loss incurred by heat transfer, κ , evaluated from pressure records by the use of equations 10 and 13, are shown in Fig. 7, plotted in (a) with respect to x_F and in (b) with respect to the progress parameter for time, τ . It is on this basis that the curves portrayed in Fig. 4 have been obtained.

The efficiencies of fuel consumption in the dynamic stage, $\eta_F \equiv M_{FC}/M_{FS}$ (ratio of the mass of fuel consumed then to that contained in the system), listed at the bottom of Table 1, were relatively low. Moreover, they were smaller when combustion rates were higher. Their restricted magnitude must have been caused by quenching at the walls and corners of the cylindrical enclosure, as well as in tubes used to fill it with test gases. The apparent anomaly of their inverse dependence on the rates of combustion was evidently due to the fact that the mixture in the test chamber was homogeneous, whereas the exothermic reaction was stratified. Faster reaction rates concomitantly with higher pressure peaks were obviously associated with diminished energy loss

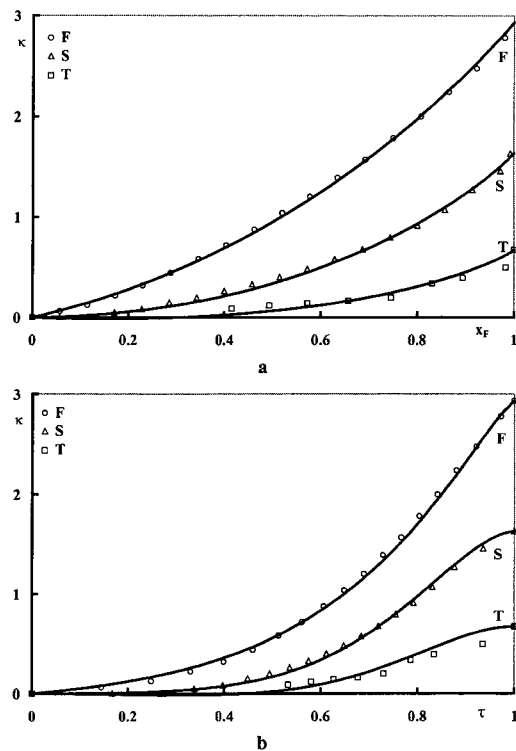


FIG. 7. Normalized profiles of energy lost by heat transfer to the walls, κ , deduced from heat transfer measurements, expressed by data points, in comparison to analytical results, evaluated from pressure records, presented by curves, as functions of (a) the progress parameter for mass fraction of fuel consumed in the course of the dynamic stage of combustion, x_F , and (b) the progress parameter for time, τ .

achieved by having the reaction zone kept further away from the walls. That had to leave more unburned mixture at the walls.

It is of interest to note that the corresponding volumetric efficiencies

$$\eta_V \equiv \frac{V_{FC}}{V_{FS}} = \frac{\alpha_f \eta_F}{1 + (\alpha_f - 1) \eta_F} \quad (14)$$

where V denotes volume, while $\alpha_f \equiv v_P/v_R$ is the ratio of specific volumes of products and reactants, listed below η_F , are significantly higher as a consequence of the fact that the density of products is appreciably lower than that of reactants.

Thus, heat transfer data provided a measure of the efficiency with which the fuel was utilized in the course of the dynamic stage of combustion. Close agreement between the functional relationships, $\kappa(x_F)$, concomitantly with $x_p(x_F)$, obtained from heat transfer measurements and those deduced from recorded pressure traces, demonstrates the validity of

the correlation expressed by equation 13. Indeed, as a true phase relation, this correlation should be independent of the shape of the enclosure, as well as the physical conditions under which the exothermic process of combustion is executed. In particular, it cannot be affected by piston motion taking place in the physical space, especially in the vicinity of the top dead center where the exothermic process of combustion takes place in a piston engine.

Summary

We have raised the following points:

1. Recognizing the essential role played in a closed combustion system by the dynamic stage, manifested by a monotonic pressure rise, it was carved out of the ignition stage preceding it and the decay by which it is followed.
2. The study revealed that the mechanism of the dynamic stage was influenced by heat transfer to the walls, the only kind of energy loss taking place in its course.
3. The total amount of energy lost by heat transfer was deduced from time profiles of its fluxes, deduced from thin film thermometer measurements taken at strategic locations on the walls of a cylindrical enclosure simulating the environment of a piston engine, by solutions of Duhamel integrals taking advantage of the observed self-similarity between these fluxes.
4. Concomitant time profiles of thermodynamic parameters were deduced from pressure records by solutions of an inverse problem: evaluation of a process from its output, based on the balances of mass, volume, and energy.
5. Especially instrumental in this respect was the concept of the function—a biparametric expression of progress parameters for the evolution of pressure and consumption of fuel.
6. A correlation between the effective part of consumed fuel, manifested by the pressure rise, and its total amount was thus determined, permitting the profile of energy lost by heat transfer from a closed combustion system to be deduced from the measured pressure trace.
7. All the thermodynamic and dynamic features of a closed combustion system were thus evaluated, including prominently the effectiveness with which fuel was utilized in the course of its dynamic stage, as well as the size of the dynamic quench layer. The former was expressed in terms of the mass fraction of burned charge, while the latter referred to the volume fraction of unburned charge, at the instant when pressure reached its peak. Both got lower at higher combustion rates and peak pressures.

Nomenclature

Symbols

A	air
F	fuel
k	normalized slope of a locus of states on the Le Chatelier diagram
M	mass
n	polytropic coefficient
p	pressure
P	products
P	normalized pressure
R	reactants
Q	exothermic energy
q _w	energy lost by heat transfer to the walls
t	time
U	normalized internal energy
V	volume
w	Pv, thermodynamic reference parameter
W	normalized thermodynamic reference co-ordinate
Y	mass fraction of products
x _k	dependent variable of the life function
α	coefficient of the life function
ζ	power index of the life function
η _F	efficiency of the use of fuel for the dynamic stage of combustion
κ	normalized energy lost by heat transfer to the walls
ν	specific volume
χ	power index of life function

Subscripts

C	charge
f	final
F	fuel
hp	specific enthalpy and pressure at initial state
i	initial
k	P, F
p	effective part of fuel consumed to generate energy
P	products
R	reactants
q	ineffective part of fuel expended for heat transfer to the walls
S	system
uv	specific internal energy and specific volume at initial state

Superscripts

'	derivative with respect to time
"	derivative with respect to surface area

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